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Minimizing Rental Cost under Specified Rental Policy in Two Stage Open shop Scheduling Problem

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Abstract

This Paper is an attempt to study two stage open shop scheduling problem in which processing time of the jobs are given. The objective of the paper is to obtain an algorithm to minimize the rental cost of the machines in two stage open shop scheduling problem under specified rental policy. A numerical illustration is given to justify the proposed algorithm.

Keywords: Open shop scheduling, Rental policy, Processing time, Utilization time, completion time, Elapsed time.

Introduction

Scheduling is an important process widely used in manufacturing, production, management, computer science so on. Appropriate scheduling can reduce rental cost of machines and running time of machines. Optimal schedule for given sets of jobs can help's supervisors to effectively control job flows and provide valuable result. The earliest result for flowshop scheduling problem was introduced by Johnson [11] in order to minimize the total idle time of the machines. Further the work was developed by Jackson J.R [10], Maggu and Das [12], Yoshida and Hitomi [17], D.Rebaine[4], Anup[1], Gupta Deepak and singh T.P [6] by introducing different parameter such as Transportation time, breakdown interval, weightage, setup time separated from processing time etc. Open shop scheduling differ from flowshop scheduling in the sense that there is no restriction placed on the order of the machines i.e, operations can be performed in any order, first machine to second machine or second machine to first machine and not known in the advances. Maggu P.L and Harbans Lal [13] introduced the concept of n*2 Open shop scheduling problem including job-block criteria. Gupta Deepak and Singh T.P [6] studied n*2 Open shop scheduling problem to minimize the idle time of the machines in which processing times associated with their respective probabilities including job-block criteria. further work was extended by Gupta Deepak and Renuka [9] by associated transportation time and weightage of jobs. Bagga P.C and Narian L. [14] introduced the concept of two stage flowshop scheduling problem to minimize rental cost of the machines under predefined rental policy. Further Singh T.P. and Gupta Deepak [15] associated probabilities with processing time of the jobs in their studies to minimize the rental cost of machines including job-block criteria under specified rental policy. Work was extended by Gupta Deepak and Sharma S [7], Gupta D,Singla P and Sharma S [8] by introducing the concept of Breakdown interval and Set up times. Gupta Deepak and Shashi Bala [5] introduced the concept of two stage specially structured open shop problem to minimize the rental cost of machines including transportation time. In this paper,the concept of minimizing the rental cost in two stage flow shop scheduling problem made by Bagga P.C and Narian L.{14] has been applied in open shop scheduling problem under predefined rental policy. Thus the work studied here is wider applicable and more significant. Here we have developed an algorithm to minimize the rental cost of machine in two stage open shop scheduling problem.

Practical Situation

Open shop scheduling problems arise in several industrial situations. For example, consider a large aircraft garage with specialized work-centers. An airplane may require repairs on its engine and electrical circuit system. These two tasks may be carried out in any order but it is not possible to do these tasks on the same plane simultaneously. Other applications of open shop scheduling problems are in automobile repair, quality

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control centers, semiconductor manufacturing, teacher-class assignments, examination scheduling, and satellite communications etc. In the era of globalization or global uncertainties, to meet the challenges of the business, one does not always have enough funds to invest in advanced machines to update the technology. Under such circumstances the machines has to be taken on rent. Rental of machines is an affordable and quick solution for having the equipment and up gradation to new technology.

Notations

S :Sequence of jobs 1,2,3,.....,n
M_j :Machine, j=1,2,3.....
A_i :Processing time of i^{th} job on machine M_1
B_i :Processing time of i^{th} job on machine M_2
C_j :Rental cost per unit time of machine j.
$t_1(S)$:Total completion time of last job on first machine for sequence S.
$t_2(S)$:Total completion time of last job on second machine for sequence S
CT(S) :Completion time of first job on first machine for sequence S.
U(S) :Utilization time of second machine for sequence S.
R(S) :Rental cost for sequence S.

Rental Policy

The machines will be taken on rent as and when they are required and are returned as and when they are no longer required. i.e. the first machine will be taken on rent in the starting of the processing the jobs, 2^{nd} machine will be taken on rent at time when 1^{st} job is completed on the 1^{st} machine.

Problem Formulation

Let n jobs 1,2,……..,n be processed on two machines M_1 and M_2 in any order i.e. the jobs will be processed first on M_1 and then on M_2 or first on M_2 and then on M_1 under the specified rental policy . Let A_i be the processing time of i^{th} job on machine M_1 and B_i be the processing time of i^{th} job on machine M_2 Our aim is to find the optimal or near optimal sequence $\{S_k\}$ of the jobs which minimize the rental cost of machines.

Jobs	Machine M_1	Machine M_2
i	A_i	B_i
1	A_1	B_1
2	A_2	B_2
3	A_3	B_3
-	-	-
n	A_n	B_n

TABLEAU-1

Mathematically, the problem is stated as:

Minimize $R(S_k) = \sum_{i=1}^{n} A_i(S_K) \times C_1 + U_2(S_K) \times C_2$

Subject to constraint: Rental Policy(P).

i.e, Our objective is to minimize rental cost of machines under pre-defined rental policy.

Assumption

1. Two jobs cannot be processed on a single machine at a time.
2. Jobs are independent to each other.
3. Per-emption is not allowed i.e. once a job started on a machine, the process on that machine cannot be stopped unless the job is completed.
4. Let n jobs be processed through two machines M_1 and M_2 in order M_1 to M_2 and in order M_2 to M_1.
5. Machine break down is not considered.

Algorithm

Step1:For order $M_1 \rightarrow M_2$

Construct a Set S_A of all processing time A_i where $A_i \leq B_i$ and Set S_A' of all processing time A_i where $A_i \geq B_i$.

Step 2: Let S_1 denote a sub optimal sequence of jobs corresponding to non-decreasing time in set S_A ,Similarly S_1' corrosponding to set S_A'.

Step 3: The augmented ordered sequence $J_1 = (S_1, S_1')$ are optimal/or near optimal sequence for processing the jobs on machine M_1.

Step 4: For order $M_2 \rightarrow M_1$

Construct a Set S_B of all processing time B_i where $B_i \leq A_i$ and Set S_B' of all processing time B_i where $B_i \geq A_i$.

Step 5: Let S_2 denote a sub optimal sequence of jobs corresponding to non-decreasing time in set S_B ,Similarly S_2' corrosponding to set S_B'.

Step 6: The augmented ordered sequence $K_1 = (S_2, S_2')$ are optimal/or near optimal sequence for processing the jobs on machine M_2.

Step 7: Observe the processing time of 1^{st} job of J_1 on the first machine i.e, on M_1. Let it be α. Similarly observe the processing time of 1^{st} job of K_1 on the first machine i.e, on M_2. Let it be β .

Step 8: Obtain all the jobs having processing time on M_1 greater than α. Put these jobs one by one in the first position of the sequence J_1.Keeping other jobs of the sequence J_1 in same order. Let these sequence be J_2 ,J_3....J_r. Similarly for order $M_2 \rightarrow M_1$ sequences are K_2 , K_3 ....K_s .

Step 9: Prepare in-out table for each sequence J_p (p=1,2,3....r) for order $M_1 \rightarrow M_2$ and K_q (q=1,2,3...s) for order $M_2 \rightarrow M_1$ and evaluate total completion time of last job of each sequence i.e, $t_1(J_p)$ & $t_2(J_p)$ on machine M_1 &M_2 respectively. Similarly $t_1(J_q)$ & $t_2(J_q)$ on machine M_2 &M_1 respectively.

Step 10: Evaluate completion time $CT(J_p)$ of first job of each sequence J_p on machine M_1 for order $M_1 \rightarrow M_2$, Similarly $CT(K_q)$ of first job of each sequence K_q on machine M_2 for order $M_2 \rightarrow M_1$ respectively.

Step 11: Compute utilization time U_p and U_q of second machine as follow:

$U_p = t_2(J_p) - CT(J_p)$ p= 1,2,3.....r for order $M_1 \rightarrow M_2$

$U_q = t_2(K_q) - CT(K_q)$ q = 1,2,3.....s for order $M_2 \rightarrow M_1$

Step 12: Find min (U_p), p=1,2,3....r. let it be corresponding to job p=m , then J_m is the optimal sequence for minimum rental cost for order $M_1 \rightarrow M_2$. similarly Find min (U_q), q=1,2,3....s. let it be corresponding to job q=n , then k_n is the optimal sequence for minimum rental cost for order $M_2 \rightarrow M_1$

Step 13: Find rental cost for sequence J_m and K_n as follow:

$$R(J_m) = t_1(J_m) \times C_1 + U_m \times C_2$$

$$R(k_n) = t_1(K_n) \times C_2 + U_n \times C_1$$

Where C_1 and C_2 are the rental cost per unit time of machine M_1 and M_2 respectively.

Numerical illustration

Consider 5 jobs and 2 machines open shop problem to minimize the rental cost. The Processing time of these jobs are giveb as follow. Rental cost per unit time for machine M_1 and M_2 are 10 and 15 respectively.

Jobs	Machine M_1	Machine M_2
i	A_i	B_i
1	22	18
2	28	30
3	32	16
4	17	21
5	20	25

TABLEAU-2

Our objective is to obtain optimal schedule and order of machines to minimize the rental cost of the machines,under the rental policy P.

Solution: For order $M_1 \rightarrow M_2$

Jobs	Machine M_1	Machine M_2
i	A_i	B_i
1	22	18
2	28	30
3	32	16
4	17	21
5	20	25

TABLEAU-3

As per step 1:

Construct a set S_A and S_A'

S_A = { 28,17,20 } S_A' = { 22,32 }

As per step 2:

S_1 = { 4,5,2 } S_1'={ 1,3 }

As per step 3:

Augmented ordered sequence

$J_1 = (S_1, S_1')$

J_1 = { 4,5,2,1,3 }

As per step 7, 8:

Other optimal sequences

J_2 =(5,4,2,1,3)

J_3 =(1,4,5,2,3)

J_4 =(2,4,5,1,3)

J_5 =(3,4,5,2,1)

As per step 9,10 and 11:

In-out table for sequence J_r (r=1,2,3,4,5) as follow:

For J_1 = 4-5-2-1-3

Jobs	Machine M_1	Machine M_2
4	0-17	17-38
5	17-37	38-63
2	37-65	65-95
1	65-87	95-113
3	87-119	119-135

TABLEAU-4

Total elapsed time on machine $M_1 = t_1(J_1)$ =119

Total elapsed time on machine $M_2 = t_2(J_1)$ =135

Utilization time of second machine $U(J_1)$ =135-17

= 118

For J_2 = 5-4-2-1-3

Jobs	Machine M_1	Machine M_2
5	0-20	20-45
4	20-37	45-66
2	37-65	66-96
1	65-87	96-114
3	87-119	119-135

TABLEAU-5

Total elapsed time on machine $M_1 = t_1(J_2)$ =119
Total elapsed time on machine $M_2 = t_2(J_2)$ =135
Utilization time of second machine $U(J_2)$ =135-20
= 115

For J_3 = 1-4-5-2-3

Jobs	Machine M_1	Machine M_2
1	0-22	22-40
4	22-39	45-61
5	39-59	61-86
2	59-87	87-117
3	87-119	119-135

TABLEAU-6

Total elapsed time on machine $M_1 = t_1(J_3)$ =119
Total elapsed time on machine $M_2 = t_2(J_3)$ =135
Utilization time of second machine $U(J_3)$ =135-22
= 113

For J_4 = 2-4-5-1-3

Jobs	Machine M_1	Machine M_2
2	0-28	28-58
4	28-45	58-79
5	45-65	79-104
1	65-87	104-122
3	87-119	122-138

TABLEAU-7

Total elapsed time on machine $M_1 = t_1(J_4)$ =119
Total elapsedtime on machine $M_2 = t_2(J_4)$ =138
Utilization time of second machine $U(J_4)$ =138-28
= 110

For J_5 = 3-4-5-2-1

Jobs	Machine M_1	Machine M_2
3	0-32	32-48
4	32-49	49-70
5	49-69	70-95
2	69-97	97-127
1	97-119	127-145

TABLEAU-8

Total elapsed time on machine $M_1 = t_1(J_5)$ =119
Total elapsed time on machine $M_2 = t_2(J_5)$ =145
Utilization time of second machine $U(J_5)$ =145-32
= 113

As per step 12:
Total utilization time of machine M_1 is fixed 119 units and minimum utilization time of machine M_2, $U(J_4)$ = 110 units i.e, for sequence J_4 =2-4-5-1-3
So optimal sequence is J_4 =2-4-5-1-3

As per step 13:
Mnimum rental cost R(J)= 119 ×10+ 110×15
=2840

Order $M_2 \rightarrow M_1$

Jobs	Machine M_2	Machine M_1
i	B_i	A_i
1	18	22
2	30	28
3	16	32
4	21	17
5	25	20

TABLEAU-9

As per step 4 and 5:
Construct a set S_B and S_B'
S_B = { 18,16 } S_B' = { 30,21,25 }
S_2 = { 3,1 } S_2'={ 4,5,2 }

As per step 6:
Augmented ordered sequence
K_1 =(S_2 , S_2')
K_1 = { 3,1,4,5,2 }

As per step 7, 8:
Other optimal sequences
K_2 =(1,3,4,5,2)
K_3 =(4,3,1,5,2)
K_4 =(5,3,1,4,2)
K_5 =(2,3,1,4,5)

As per step 9,10 and 11:
In-out table for sequence K_s (s=1,2,3,4,5) as follow:

For k_1 = 3-1-4-5-2

Jobs	Machine M_2	Machine M_1
3	0-16	16-48
1	16-34	48-70
4	34-55	70-87
5	55-80	87-107
2	80-110	110-138

TABLEAU-10

Total elapsed time on machine $M_2 = t_1(k_1)$ =110
Total elapsed time on machine $M_1 = t_2(k_1)$ =138
Utilization time of second machine $U(k_1)$ =138-16
= 122

For k_2 = 1-3-4-5-2

Jobs	Machine M_2	Machine M_1
1	0-18	18-40
3	18-34	40-72
4	34-55	72-89
5	55-80	89-104
2	80-110	110-138

TABLEAU-11

Total elapsed time on machine $M_2 = t_1(k_2)$ =110
Total elapsed time on machine $M_1 = t_2(k_2)$ =138
Utilization time of second machine $U(k_2)$ =138-18
= 120

For k_3 = 4-3-1-5-2

Jobs	Machine M_2	Machine M_1
4	0-21	21-38
3	21-37	38-70
1	37-55	70-92
5	55-80	92-112
2	80-110	112-140

TABLEAU-12

Total elapsed timeon machine $M_2 = t_1(k_3)$ =110
Total elapsed time on machine $M_1 = t_2(k_3)$ =140
Utilization time of second machine $U(k_3)$ =140-21
= 119

For k_4 = 5-3-1-4-2

Jobs	Machine M_2	Machine M_1
5	0-25	25-45
3	25-41	45-77
1	41-59	77-99
4	59-80	99-116
2	80-110	116-144

TABLEAU-13

Total elapsed time on machine $M_2 = t_1(k_4)$ =110
Total elapsed time on machine $M_1 = t_2(k_4)$ =144
Utilization time of second machine $U(k_4)$ =144-25
= 119

For k_5 = 2-3-1-4-5

Jobs	Machine M_2	Machine M_1
2	0-30	30-58
3	30-46	58-90
1	46-64	90-112
4	64-85	112-129
5	85-110	129-149

TABLEAU-14

Total elapsed time on machine $M_2= t_1(k_5)$ =110
Total elapsed time on machine $M_1 = t_2(k_5)$ =149
Utilization time of second machine $U(k_5)$ =149-30
= 119

As per step 12:
Total utilization time of machine M_2 is fixed 110 units and minimum utilization time of machine M_1, 119 units i.e, for sequence k_3 =4-3-1-5-2, K_4 =5-3-1-4-2, K_5 =2-3-1-4-5
So optimal sequence is K_3 , K_4 , K_5
As per step 13:
Mnimum rental cost $R(K)= 110 \times 15+ 119\times 10$
=2840

Total rental cost when the order is from M_1 to M_2 for the sequence J_4 is 2840 units and for the sequence K_3 ,K_4 ,K_5 is also 2840 when order is M_2 to M_1. Hence both order gives the same rental cost which is minimum.

Remark

1. The work may further be exrented for n jobs 3 machines open shop problem.
2. The study can further be extended by considering various parameters such as transportation time, job-block, setup time, breakdown intervals etc

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